SCATTERING CHARACTERISTICS OF A BEAM MODE IN DIELECTRIC-SLAB OPTICAL WAVEGUIDE

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Abstract

The amount and directivity of scattering light in dielectric-slab optical waveguides, produced in a propagating beam mode due to refractive-index inhomogeneities of the thin film and due to boundary irregularities of film-substrate and film-air interfaces are calculated by means of a perturbation method together with the use of a stationary phase method and obtained as a function of their correlation length and variance, thin film thickness, and refractive index difference.

There are two kinds of fluctuations which cause mode conversion and radiation loss in dielectric-slab optical waveguides. They are random irregularities of the film-air and film-substrate interfaces[1,2] and random fluctuations of the refractive index in the thin film region (core region) of a slab waveguide.[3,4] In this paper, when the dominant $TE_0$ mode is incident onto an asymmetric slab waveguide with such imperfections, the radiated power into external regions of the air and the substrate are evaluated by means of a perturbation technique together with the use of a stationary phase method. For simplicity, the boundary deviation from perfect straight wall is assumed to be small compared with the film thickness and the local change of the refractive index is assumed to be uncorrelated with imperfections of the interface. Therefore, the radiated power of scattering light due to both the refractive-index inhomogeneities and the boundary wall irregularities can be independently estimated and superposed on an ensemble average.

The waveguide to be studied here is shown in Fig.1 and consists of three layers which are air, thin film and substrate having refractive indices of $n_1$, $n_2$ and $n_3$. The relation of $n_1 < n_3 \lesssim n_2$ is assumed to exist in this waveguide. The waveguide with wall imperfections is mathematically described by a square of its refractive index and given by

$$n^2(x,z) = n_i^2 + \Delta n^2(x,z) \tag{1}$$

where

$$n_i^2 = \begin{cases} n_1^2 & ; \ x \geqq 0 \\ n_2^2 & ; \ -t \leqq x \leqq 0 \\ n_3^2 & ; \ x \leqq -t\,. \end{cases} \tag{2}$$

The additional term $\Delta n^2$ describes a geometrical deviation from the perfect shape of the waveguide and is given by

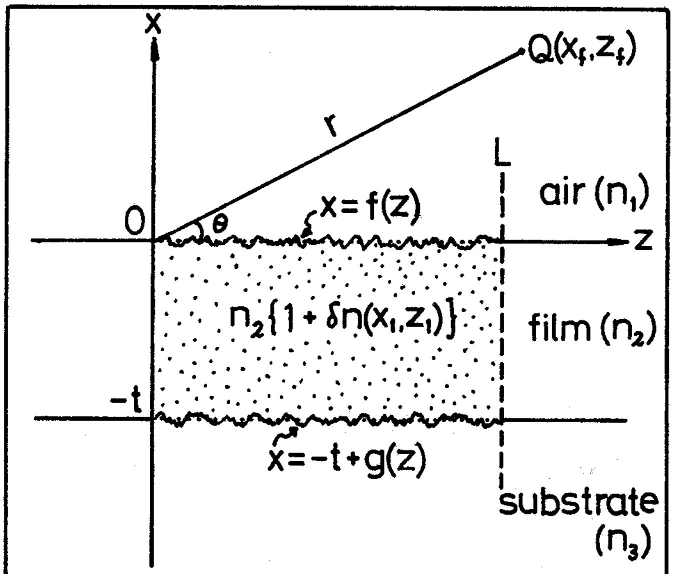


Fig.1 Coordinates of a dielectric-slab optical waveguide with refractive index inhomogeneities and waveguide wall irregularities.

$$\Delta n^2(x,z) = \begin{cases} n_2^2 - n_1^2 & ; \ x \geqq 0 \text{ and } x \leqq f(z) \\ n_1^2 - n_2^2 & ; \ x \leqq 0 \text{ and } x \geqq f(z) \\ n_3^2 - n_2^2 & ; \ x \geqq -t \text{ and } x \leqq -t + g(z) \\ n_2^2 - n_3^2 & ; \ x \leqq -t \text{ and } x \geqq -t + g(z) \\ 0 & ; \ \text{otherwise} \end{cases} \tag{3}$$

where f(z) and g(z) are random functions of z. On the other hand, the local change of the refractive index in the core region is given by

$$n^2(x,z) = \begin{cases} n_1^2 & ; \ x \geqq 0 \\ n_2^2 + 2n_2\delta n(x,z) & ; \ -t \leqq x \leqq 0 \\ n_3^2 & ; \ x \leqq -t \end{cases} \tag{4}$$

where $\delta n$ is a random function of x and z. Here, any imperfection of the waveguide is assumed to exist in the region between z = 0 and z = L where the perturbed waveguide is connected to an ideal waveguide. The random functions of $\delta n$, f(z) and g(z) obey the following statistical restrictions:

$$|\delta n(x,z)| \ll 1, \quad \langle \delta n(x,z) \rangle = 0$$

$$\langle \delta n(x_1,z_1)\, \delta n(x_2,z_2) \rangle = V^2 \exp\left\{ -\frac{|x_1-x_2|}{a_x} - \frac{|z_1-z_2|}{a_z} \right\}$$

$$|f(z)|, \; |g(z)| \ll t, \quad \langle f(z) \rangle = \langle g(z) \rangle = 0$$

$$\langle f(z_1) f(z_2) \rangle = V_f^2 \exp\left\{ -\frac{|z_1-z_2|}{b_f} \right\} \tag{5}$$

$$\langle g(z_1) g(z_2) \rangle = V_g^2 \exp\left\{ -\frac{|z_1-z_2|}{b_g} \right\}$$

$$\langle f(z_1) g(z_2) \rangle = C^2 \exp\left\{ -\frac{|z_1-z_2|}{b_z} - \frac{t}{b_x} \right\}$$

where $\langle \cdots \rangle$ denotes an ensemble average and $V^2$, $V_f^2$ and $V_g^2$ are the variances of $\delta n$, f(z) and g(z), and $C^2$ means the covariance of f(z) and g(z). $a$ and b involved in the exponential term of Eq.(5) represent the correlation length for the refractive index inhomogeneities and the wall imperfections, respectively.

Since the imperfections of refractive index inhomogeneities and waveguide wall irregularities produce a radiating perturbed field in the propagating $TE_0$ mode, the perturbed electric field $\delta E(x_f,z_f)$ in the air region is given by a sum of the fields due to these imperfections and expressed by

$$\begin{aligned}
\delta E_y^{(\pm)}(x_f,z_f) = &-\frac{j n_2^2 k^2}{2\omega\mu} \sum_{i=1}^{2} \Bigg[ \int_0^L \int_{-t}^0 \delta n(x_1,z_1) E_0(x_1,z_1) \\
&\times \left\{ \int_0^{n_1 k} E_{ai}(x_f,z_f,\beta) E_{ai}^*(x_1,z_f,\beta) \exp\left(\mp j\beta(z_f-z_1)\right) \frac{\beta}{\rho}\, d\beta \right\} dz_1\, dx_1 \Bigg] \\
&- \frac{jk^2}{2\omega\mu} \sum_{i=1}^{2} \int_0^L \Bigg[ (n_2^2-n_1^2) E_0(0,z_1) f(z_1) \\
&\times \int_0^{n_1 k} E_{ai}(x_f,z_f,\beta) E_{ai}^*(0,z_f,\beta) \exp\left(\mp j\beta(z_f-z_1)\right) \frac{\beta}{\rho}\, d\beta \\
&- (n_2^2-n_3^2) E_0(-t,z_1) g(z_1) \\
&\times \int_0^{n_1 k} E_{ai}(x_f,z_f,\beta) E_{ai}^*(-t,z_f,\beta) \exp\left(\mp j\beta(z_f-z_1)\right) \frac{\beta}{\rho}\, d\beta \Bigg] dz_1
\end{aligned} \tag{6}$$

In the equation, the asterisk denotes a complex conjugate and $E_m(x,z)$ and $E_{ai}(x,z,\beta)$ are the electric fields of the guided $TE_m$ mode and TE radiation mode. $k = 2\pi/\lambda$, $\omega$ and $\mu$ are the wavenumber of light, the angular frequency, and the permeability in vacuum, respectively. $\beta$ and $\rho$ represent the propagation constants in the z and x directions for the substrate region. The plus and minus signs in the superscript in the equation correspond to the forward and backward scatterings. The integral with respect to $\beta$ appeared in Eq.(6) may be performed by means of a stationary phase method. The time-average power $P(\theta)$ flowing into the unit area perpendicular to $\overline{OQ}$ through the observation point $Q(x_f,z_f)$ is expressed by using a Pointing vector $\mathbf{S}$ and given by

$$P(\theta) = \mathrm{Re}\,[\mathbf{S}\cdot\mathbf{i}_r]\,r \tag{7}$$

where

$$\mathbf{S} = \frac{1}{2}\,\delta\mathbf{E}(x_f,z_f) \times \delta\mathbf{H}^*(x_f,z_f) \tag{8}$$

Re denotes a real part, and $\mathbf{i}_r$ is a unit vector in the radial direction. In Eq.(8), the vector components of the perturbed magnetic field $\delta\mathbf{H}(x_f,z_f)$ are obtained by using the perturbed electric field $\delta E_y(x_f,z_f)$ and given by

$$\begin{aligned}
\delta H_x^{(\pm)}(x_f,z_f) &= \frac{1}{j\omega\mu} \frac{\partial E_y^{(\pm)}(x_f,z_f)}{\partial z_f} \\
\partial H_z^{(\pm)}(x_f,z_f) &= -\frac{1}{j\omega\mu} \frac{\partial E_y^{(\pm)}(x_f,z_f)}{\partial x_f}.
\end{aligned} \tag{9}$$

Using Eqs.(7)-(9), we can evaluate the radiated power caused by both the refractive index inhomogeneities and the waveguide wall irregularities and obtain the radiation pattern as a function of the angle $\theta$. Following exactly the same procedure as in the air region, we can also obtain the time-average power of radiation in the substrate region.

References


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